Surface Charge Distributions in Circuits

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What drives current?

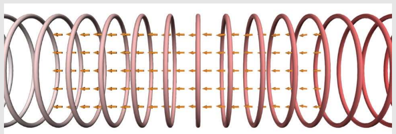
- The fields produced by a battery or capacitor polarize circuit conductors. The resulting surface charges contribute additional fields, leading rapidly to a steady state in which the net field inside a wire is parallel to the wire and constant along the wire and across its cross section, as predicted by the Kirchhoff node and loop rules.
- Similarly, in electrostatics, external fields applied to an isolated conductor polarize the conductor, leading rapidly to an equilibrium state, with a net field that is zero inside the conductor.

In these diagrams, red represents positive surface charge, blue is negative, and gray is neutral.

Slightly positive

More positive

Strongly positive



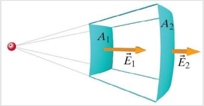
A constant gradient of charge on a line of rings produces a remarkably uniform electric field inside the rings, like the field inside circuit wires.

Computing a Surface Charge Distribution

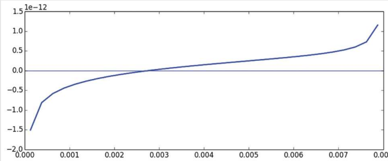
- Divide the surface into N small square tiles and calculate the field produced at the center of the ith tile that would be contributed by the jth tile, assuming that every tile carries a point charge of 1 C at its center. Take the dot product with the tile's outward-going normal, and multiply by $\sigma A \Delta t$, where σ is the conductivity, A is the area of the tile, and Δt is the time step in the relaxation algorithm. In an N by N matrix M, set M_{ij} to the calculated quantity.
- Divide one tile into a large number of sub-tiles and calculate the field just inside the tile due to a uniformly distributed charge of 1 C on the tile, and place this quantity in the M_{ii} diagonal positions of the matrix.
- For the ith tile, compute the field E just inside each tile due solely to the fixed external charges, take the dot product of E with the outward-going normal, and multiply by $\sigma A \Delta t$. In a list X that is N long, set X_i to this quantity. Now the iterative relaxation procedure can begin.
- For the ith tile, calculate the sum of $M_{ij}q_j$ for j running from 1 to N, where q_j is the charge on the jth tile, and add the contribution of the fixed charges, X_i. Add this quantity to the present charge of the ith tile, as this sum represents the charge transfer toward (or away from) this tile due to the net electric field contributed by all the sources, including its own charge. (A technical point: save this change in the charge and update all the tile charges at the end of the time step rather than updating the tile charge immediately, which would distort its role in affecting the other tiles.)
- Repeat these computations until the charge on one or more representative tiles is no longer changing significantly. This gives the final surface charge distribution.

This relaxation algorithm can be applied to DC circuits (or RC circuits for short times) as well as to electrostatic situations. In the electrostatic case one finds that the net field indeed goes to zero inside the conductor. In the case of a DC circuit the net field inside the conductors should be parallel to the conductors and uniform along and across a conductor of constant cross section and uniform conductivity, and this is found to be the case. The computed surface charge distributions were checked by using them to calculate and display the electric field throughout the region of interest. This procedure is a modified version of that used by Norris Preyer in his article "Surface charges and fields of simple circuits", Am. J. Phys. 68 (11) 1002-1006, Nov. 2000. The main differences are that internal currents are ignored (see explanation on the right), and 3D visualization was available using VPython (vpython.org, glowscript.org). The work also benefited from a huge increase in computer speed in the last 15 years.

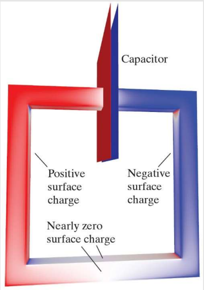
A positive charge on the left polarizes a metal block, giving rise to a distribution of charge on the surface of the metal. The vector sum of the electric fields contributed by the external charge and the surface charges is zero inside the metal.



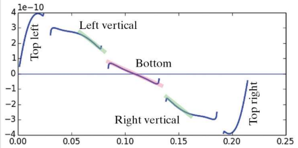
It is unnecessary to calculate fields and currents in the interior of a conductor because Gauss's law shows that the electric flux through any interior volume is zero, and hence the net flux of current through the volume is also zero. The conductor starts out with no charge, neither in the interior nor on the surface, and at each iteration the surface charges change but not the zero interior volume charge.



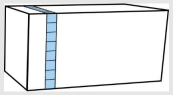
A graph of the ring charge along the polarized block of metal shown above. Along much of the block there is a nearly constant gradient of surface charge, with charge buildup on the ends.

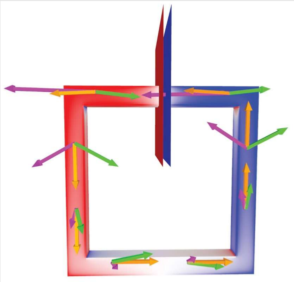


A simple circuit shows a roughly constant gradient of surface charge along the wires.



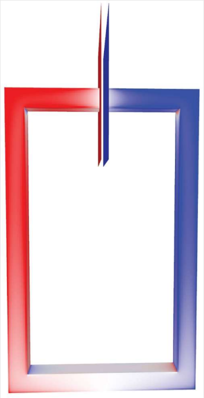
A graph of the surface charge density (C/m²) along the wire in the simple circuit shown above, averaged over a (square) ring drawn around the wire, as shown below, to average out transverse polarization that does not affect circuit behaviour. There is a roughly constant gradient of rings of surface charge along the wire. The graph omits the complications at the corners.

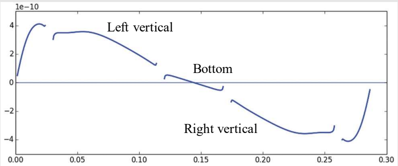




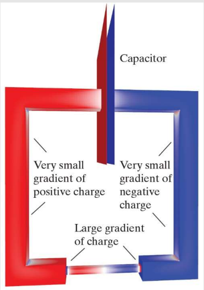
For this simple circuit, orange arrows represent the net field along the centerline of the wire, magenta arrows represent the field contributed by the capacitor, and green arrows represent the field contributed by all of the surface charges.

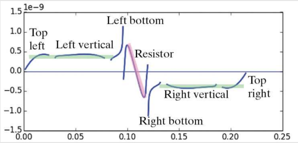
Note that along the sections of wire next to the capacitor the gradient of surface charge in the graph on the left goes the "wrong way". As can be seen above, the fringe field of the capacitor is significantly larger than the net field in the wire, and charge buildup at the corners leads to the field contributed by the surface charge pointing to the right (represented by the green arrows). Along the bottom wire the net field is due almost entirely to a nearly constant gradient of rings of surface charge.



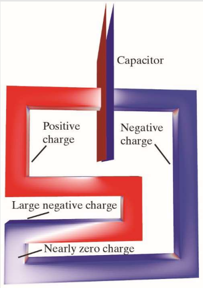


With longer wires, the nearly constant gradient of rings of surface charge is even more apparent.





A thin section of wire acts as a resistor. Charge buildup at the ends of the thin wire, visible in the graph, leads to a negligible gradient of surface charge along the thick wire and a very large gradient of surface charge along the thin wire, producing the large field necessary to drive the same amount of current through the thin wire as flows through the thick wire.



A more complicated circuit geometry, in which the surface charge on a section of wire strongly polarizes transversely a neighboring wire.

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Surface Charge Program

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