Modern mechanics

Ruth W. Chabay[a] and Bruce A. Sherwood
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695

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We consider the goals of the introductory course in classical mechanics taken by physics majors and argue both that these goals are not well met in actual courses and that the goals themselves should be rethought. We propose alternative goals and describe an introductory ''modern mechanics'' course that addresses these alternative goals. Included in the description are several genres of homework problems that are nearly absent from traditional mechanics courses at both the introductory and intermediate levels. The intermediate mechanics course could be restructured to exploit a broader foundation laid by the introductory course.

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I. WHY TEACH CLASSICAL MECHANICS?

The curriculum for undergraduate physics majors normally includes one semester of classical mechanics at the introductory level and one or two semesters at the intermediate level. Often there are additional required courses in mathematical methods which also emphasize classical mechanics. It is worth asking why so much required course work is devoted to a subject that is outside the mainstream of most contemporary physicists' interests. To begin to address this question, it is appropriate to examine the goals of the introductory mechanics course.

A. Goals of the introductory mechanics course

The goals physicists typically express for the introductory calculus-based mechanics course include learning systematic problem solving, gaining skill in using mathematics in an applied context, and learning the analytical value of separating the world into system and surroundings. Higher-level goals may include instilling in students a sense of the unity of physics, and communicating the value of a reductionist approach to understanding the world. At a more specific level, it is believed that the important concepts of momentum, energy, and angular momentum are best introduced in the context of classical mechanics even though they have wide applicability in later courses. The laboratory component of the course has varied goals, often not well articulated, ranging from illustrating the theory to learning error analysis.

Sometimes instructors state a goal of ''teaching students to think'' in general, but this goal is vague and unrealistic. First, this goal is deeply patronizing. Students come to us already capable of thinking; what we can offer are particular analytical tools peculiar to the physicist's approach. Moreover, much research (and everyday experience) has shown that transfer of general skills from one domain to another is difficult to achieve even when this difficulty is explicitly recognized and addressed. Many years ago the goal of teaching how to think was stated as the rationale for requiring the study of Latin, but there is no evidence that Latin ever had such an impact. Physics is no more likely to achieve this, and indeed it is not clear that physicists who are skilled in analyzing freebody diagrams are particularly insightful or analytical when thinking about problems in personal relations or politics.

B. The student view of the introductory course

It may well be the case that the parsimony and rich predictive power of classical mechanics are foremost in the minds of many physics instructors, but the typical student's final view of the introductory course is quite different. It is common for students to see mechanics not as an example of reductionist power, but rather as a disconnected jumble of many special purpose formulas. Problem-solving techniques appear to students as a collection of weak methods that include hunting for formulas with familiar symbols in them, or matching to similar worked-out examples. Problems that do not map directly onto previously solved problems are viewed as insoluble.

The traditional introductory mechanics curriculum shapes and reinforces this deeply flawed student view, despite the good intentions of textbook authors and teachers. Concepts that should be seen as primary (momentum, energy, angular momentum) are introduced late in the semester, and consequently do not appear central to the enterprise. An overemphasis on the kinematics of constant acceleration (with no mention of causes or interactions) leads students to see $x=(1/2)at^2$ as by far the most important and fundamental equation in all of mechanics. The power of fundamental principles is masked when students are told by the textbook to use tertiary derived equation number 6.34 to solve a problem, rather than starting the analysis from a fundamental principle.

At a more global level, the unity of physics is obscured, rather than revealed, by the treatment of classical mechanics as a closed system, with no mention of connections to thermal physics, relativistic mechanics, or quantum mechanics. A perceptive math major complained, after taking a traditional mechanics course, that he felt betrayed when he discovered (in our E&M course) that classical mechanics had limitations. He commented that classical mechanics had been presented to him as a closed axiomatic system of universal validity, and remarked that such an approach was appropriate in mathematics, but not in physics.

Many students see the physics they learn in the introductory mechanics course as unrelated to the real world, and applicable only in a special ''physics'' world of rigid objects on frictionless surfaces connected by massless strings in an airless environment. It is easy to see how students can develop this viewpoint, because they are not asked to participate in the process of modeling complex, real-world systems by making simplifications, idealizations, approximations, estimates, and selecting a fundamental principle from which to start. Nor are they asked to assess the consequences of this idealization by comparing their results to the behavior of the actual, messy, real-world system of interest. Rather, ''problem solving'' typically comes down to plugging numbers into formulas applicable to idealized systems. Physicists make good use of overly simplified models, but they are keenly aware of the idealization process, and the attendant limitations on the validity of the results. None of these aspects of physical modeling come through to the student.

C. Alternative goals for the introductory course

In light of these issues, it is important to ask what should be the nature of the introductory mechanics course taken by science and engineering students. In order to structure a curriculum whose intellectual coherence is evident to students as well as instructors, it is necessary to have a clear set of goals to guide decisions about what topics to include, and when and how to present them. We advocate the following: The goal of an introductory physics course should be to involve the student in an enterprise central to contemporary physics: the attempt to explain or predict a broad range of real physical phenomena, based on a very small number of powerful fundamental principles. In order to address this goal:

- The curriculum must require the students themselves to engage in the process of constructing models, including simplifying and idealizing messy, complex, real-world systems, making approximations, making simplifying assumptions, and estimating quantities.
- The curriculum must lead students to perceive clearly that there are only a small number of fundamental mechanics principles (the momentum principle, the energy principle, and the angular momentum principle), all of which can be applied to a very broad range of systems, over a large range of scales (galaxies to subatomic particles). Instruction must teach students to start all analyses by applying one or more of these principles.
- Twentieth century physics, especially atomic-level models of matter (but also astrophysical models) must play a central role in the course. Students need to encounter the limitations of classical models, as well as to see the power of classical and semiclassical models even at a microscopic scale.

None of these criteria is met by traditional introductory physics textbooks, which on the whole differ from each other in only minor aspects. As a result, it is very difficult to introduce any of these elements into introductory courses in a serious way; the lack of support from instructional materials severely handicaps instructors and students who wish to go beyond the standard, sanitized curriculum. Almost all problems in standard introductory (and intermediate) level mechanics texts involve systems in which all the idealization, simplification, estimation, approximation, and modeling have already been done, leaving the student to solve math problems involving anonymous 1-kg masses suspended from massless, inextensible strings on frictionless inclined planes. Open-ended prediction of the motion of interacting systems is absent from the curriculum, and only problems with closed-form analytical solutions obtainable with simple calculus are included. Macroscopic objects lack microscopic structure and are not deformable, requiring students to learn some concepts by rote (the table applies a ''normal'' force on a block because the instructor and the book said so), instead of reasoning them out [interatomic ''springs'' (bonds) are compressed by the book]. As a result, good students in traditional introductory courses become skilled at recognizing new incarnations of previously solved problems, and matching appropriate formulas to these known problem types, rather than starting from fundamental principles in analyzing systems.[1]

To be able to model complex systems and to attack problems by starting from fundamental principles, it is necessary to augment the traditional tools of algebra and calculus with the computational power of computers. Computer modeling can be seen as a benefit rather than a cost. In all fields of contemporary science and engineering, computation has become a pillar of the contemporary scientific enterprise as central as theory and experiment. We should seize the opportunity to involve students in computational science at this early stage. Computers are now fast enough that it is unnecessary to teach students complex or sophisticated approaches to numerical integration, because it is almost always possible simply to use very small step sizes without noticeable speed penalty.

There is certainly more than one way to design a curriculum which addresses these issues. Over the past decade, we have developed, taught, and refined an introductory calculus-based curriculum based on the goals listed above. In the following sections we describe this curriculum. In a final section we reflect on ways in which the intermediate mechanics course might usefully build on such an introductory course.

II. MODERN MECHANICS

The new calculus-based introductory mechanics course is embodied in the textbook *Matter & Interactions I: Modern Mechanics*[2] (Vol. II deals with electricity and magnetism). By ''modern mechanics'' we mean a course that deliberately integrates nonrelativistic and relativistic dynamics, thermal physics, and the basic quantum phenomena of energy levels and transitions between levels. This approach is obviously nonhistorical, but one that authentically represents the way contemporary physicists actually practice their science. An earlier article deals mainly with the way in which thermal physics is integrated with mechanics in the textbook.[3] Here we will focus on other aspects of the course.

Because the central feature of a curriculum is not what is presented, but what the students are asked to do, we begin with examples of homework problems in the text. We then give a detailed outline of the course to provide an overview of the intellectual and pedagogical context in which these problems are presented.

A. Examples of homework problems

We show problems that require modeling (idealization, estimation, approximation), problems that involve microscopic physical models of matter, problems drawn from astrophysics, particle physics, chemical physics, and condensed matter physics, and problems requiring the application of more than one fundamental principle. It is important to note that with few exceptions no special-purpose formulas are available, and students must reason from fundamental principles. Problems of this kind are almost entirely absent from traditional introductory (and intermediate) mechanics textbooks.

B. Macro–micro connections

Problems involving macro–micro connections engage students in explaining macroscopic phenomena with the aid of microscopic models and in understanding microscopic physics in terms of large-scale behavior. An important feature of this problem genre is that the specific composition of a material object is important, as is so often the case in the real world.

Balls and springs: A story line. A sequence of problems and activities in *Matter & Interactions*, spread over much of the semester, uses a simple ball-and-spring atomic model of a solid to explain a variety of macroscopic behaviors. First, students measure Young's modulus Y for a metal.[4] They then use Y to determine the effective stiffness k_s of the interatomic spring-like force (5 N/m for Pb, 16 N/m for Al).[5] It is then possible to model the propagation of a disturbance along a line of atoms, using k_s and the atomic mass, and compare the speed of propagation with the observed speed of sound for Pb and Al (good agreement is obtained).[6] These results for the interatomic spring stiffness can be invoked as approximate values for the interatomic spring stiffness in vibrating diatomic molecules[7] and bars of glowing iron.[8] Late in the course the students carry out statistical mechanics calculations for the Einstein model of a solid (independent quantized oscillators) using k_s and the atomic mass.[9] They fit predictions to data for the heat capacity as a function of temperature for Pb and Al and find that a good fit is consistent with the value of k_s obtained from measuring Young's modulus. The cumulative effect of these activities is to emphasize that a small number of powerful fundamental principles together with simple models of matter can explain a broad range of phenomena that seem initially to have nothing to do with each other.

C. Particle properties

The interactions of specific particles provide an intriguing and instructive alternative to considering interactions of generic objects. Such problems include the following.

(1) In the symmetric fission of uranium into two palladium nuclei, how far apart are these nuclei when they start out with zero speed? Compare with twice the radius of a palladium nucleus.[10]

(2) In a nuclear reactor, which elements make good moderators of fast neutrons? Why?[11]

(3) In the fusion reaction $p+d \rightarrow \mathrm{He}^3+\gamma$, what is the approximate input energy required to make nuclear contact, in order to make the reaction go? What is the resulting photon energy?[12]

Note that the Coulomb force and associated potential energy are encountered frequently in this mechanics course, making accessible to students a variety of interesting phenomena such as those just described. The following introductory E&M course can then build on some familiarity with electric force and energy, and can focus on the more abstract concepts of field and potential.

D. Using published data

The analysis of data obtained in actual experiments or observations provides a strong link to the real world and helps clarify the relationship between theory and experiment. Examples of such problems include the following.

(4) In 1997 the NEAR spacecraft passed within 1200 km of the asteroid Mathilde at a speed of 10 km/s relative to the asteroid. Photos transmitted by the spacecraft show Mathilde's dimensions to be about 70 km by 50 km by 50 km. It is presumably composed of rock; rock on Earth has an average density of about 5000 kg/m^3. The mass of the NEAR spacecraft is 805 kg. (a) Sketch qualitatively the path of the spacecraft. (b) Make a rough estimate of the change in momentum of the spacecraft resulting from the encounter. Explain how you made your estimate. (c) Estimate the deflection (in meters) of the spacecraft's trajectory from its original straight-line path, one day after the encounter. (d) From actual observations of the position of the spacecraft one day after encountering Mathilde, scientists concluded that Mathilde is a loose arrangement of rocks, with lots of empty space inside. What about the observations must have led them to this conclusion?[13]

(5) Given published data on orbits of stars near the center of our Milky Way galaxy obtained by infrared astronomy in the last eight years, estimate the mass of the object around which these stars orbit. Express your result in terms of solar masses. This unseen object is very compact: it must be a giant black hole.[14]

(6) In the 1911 Rutherford experiment with 10-MeV alpha particles, what was the distance of closest approach to the gold nucleus? Was there contact, which would have brought the nuclear interaction into play?[15]

(7) During an occultation of a star by Pluto in 1988 it was observed that the density of Pluto's atmosphere 50 km above the surface was 1/3 that at the surface. Spectroscopic data show that the atmosphere is mainly N_2. Estimate the temperature of Pluto's atmosphere.[16]

E. Problems requiring computer modeling

In dynamics problems involving computer modeling, the momentum principle is invoked to predict the behavior of objects or systems of objects.[17] Given the initial positions of the interacting objects, one can calculate the forces the objects exert on each other. These forces can be applied for a short time to update the momenta, and the new momenta can be used to update the positions of the objects. The student writes a simple computer program to repeat this process many times. Computers are now fast enough that it is not necessary to teach sophisticated numerical analysis techniques; simply using a very small time step provides adequate computational accuracy. The total energy of the system can be graphed as a simple check on the accuracy of the calculations. Simultaneous consideration of momentum and energy also helps students distinguish between these concepts.

The use of an appropriate programming language allows multiple representations: in addition to graphs, students create animations of the motion of physical objects. In observing the resulting simulated motion, the student has the opportunity to see the time-evolution character of the Newtonian synthesis. In these problems, kinematics is united with dynamics: changes in motion are clearly caused by interactions. Subjects include the motion of a planet around a star, including noncircular orbits,[18] a binary star (see Fig. 1),[19] a spaceship coasting to the Moon (the Ranger VII mission),[20] a spring–mass system[21] with either sliding friction, viscous friction, or air resistance,[22] projectile motion (a baseball with air resistance),[23] and the Rutherford

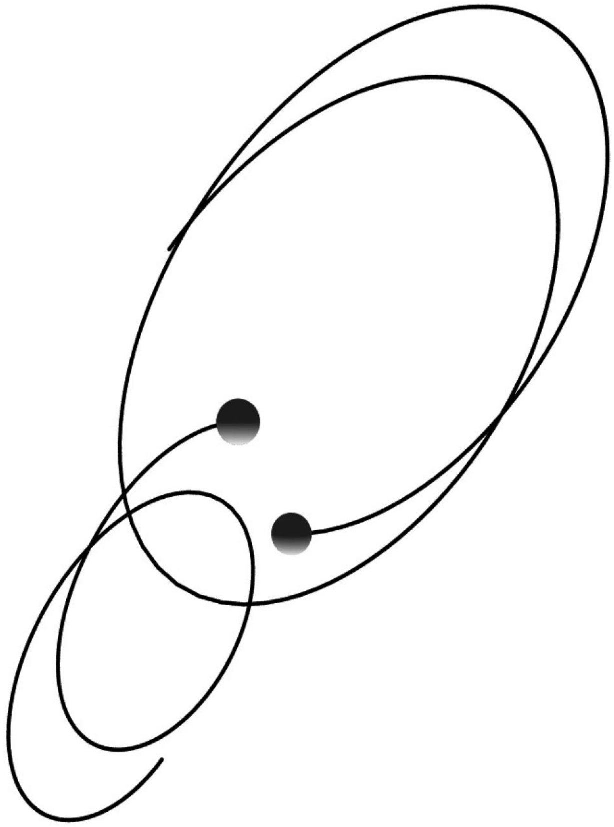

Fig. 1. A program written by a student produces a three-dimensional animation of the motion of a binary star system. No prior programming experience is required.

experiment.[24] A different kind of student program involves probability calculations that are central to the study of the statistical mechanics of the Einstein solid mentioned earlier.

III. CONCEPTUAL ORGANIZATION

In this section we describe the main elements of our modern mechanics curriculum, which is designed to develop and support student ability to tackle problems of the kinds described in the previous sections. Here is an outline of the curriculum:

Momentum
- types of matter and types of interactions
- using the momentum principle to predict future motion
- a ball-and-spring model of a solid

Energy
- energy conservation including relativistic energy
- energy in macroscopic systems including thermal energy
- energy quantization

Applications of the energy and momentum principles to multiparticle systems
- multiparticle systems and the point-particle system
- collisions including relativistic particle collisions

Angular momentum and quantized angular momentum
- applications requiring all three fundamental principles

Thermal physics
- statistical mechanics; Boltzmann factor
- kinetic theory
- heat engines

In the description that follows, the aspects of the curriculum that are closely associated with thermal physics are mentioned only briefly, because those issues have been dealt with extensively in Ref. 3.

A. Momentum

One of our primary goals is to lead students to perceive the three fundamental principles as the central ideas of introductory mechanics. To this end, we introduce the momentum principle immediately (Newton's second law), and use it as a starting point for analyses throughout the course. After a brief survey of the kinds of matter that will be analyzed (elementary particles, atoms and molecules, solids, gases, stars, the Solar system, and galaxies), we introduce the physicist's notion of ‘‘interaction'' and the ways in which interactions can be detected (change of velocity, change of identity, etc.). Momentum itself is an intuitively plausible quantity; evidently the bigger the mass of an object, the more difficult it is to change its velocity, so we expect that the change in the product $m\vec{v}$ might be related to the strength or duration of an interaction. We state that it has been found experimentally that at high speeds it is the quantity

$$\vec{p}=\frac{m\vec{v}}{\sqrt{1-v^2/c^2}} \tag{1}$$

whose change is proportional to the interaction (and we comment that this result was predicted by Einstein in the context of the theory of relativity). The momentum principle is introduced first in its difference form, for sufficiently small time interval Δt:

$$\Delta\vec{p}=\vec{F}_{\rm net}\Delta t. \tag{2}$$

This form is particularly appropriate for computer modeling. (The derivative form is discussed also, but not emphasized strongly at this point.)

Starting with momentum immediately establishes momentum and the momentum principle as centrally important concepts. (Psychological studies of memory and learning show ‘‘primacy'' to be important: people more readily remember things they have learned earlier and practiced more.[25]) Beginning with momentum addresses other goals as well. It establishes approximation as legitimate and useful: When is it okay to approximate momentum by $m\vec{v}$? Approximation is a radical idea to most beginning students, who have been told that physics is an exact science. This early introduction gives students the opportunity to use the momentum concept for an extended period (several weeks) before energy is introduced, significantly decreasing the likelihood that momentum and kinetic energy will be confused.[26–28]

The power of the momentum principle emerges as students engage in activities that would be impossible without it: predicting moment by moment the motion of several objects interacting gravitationally (planets, comets, spaceships, binary stars). The same principle is invoked to model systems interacting via other force laws (spring–mass systems, with and without friction, solids modeled as lattices of microscopic masses and springs, projectiles with air resistance). These and other activities allow practice with kinematics in context—changes in momentum occur as a result of interactions; there are no unmotivated accelerations. The integration of computer modeling as a serious component of the introductory course reflects the importance of computation in physics and other scientific and technical disciplines.

This class of problem is completely absent from most traditional textbooks, which stress the small number of analytical solutions accessible at the introductory level (projectile motion, circular motion, simple harmonic motion), or problems in which it is possible to deduce unknown forces, given known constrained motion (such as a block sliding without tumbling down an incline). As a result, students in the introductory course never see for themselves the predictive power of the momentum principle.

In contrast, we deliberately choose to spend little time on traditional dynamics problems involving freebody diagrams. Acquiring some measure of expertise with complicated freebody problems requires a large investment of time and effort, but we observe that these techniques have little direct transfer to other domains and later courses. In fact, there is almost no connection with other courses taken by physics majors. Students in some engineering disciplines do take later courses in statics and dynamics, but our engineering colleagues report that they depend very little on their students' experience in the introductory physics course, in part because the approach in the engineering course uses different notation and has different emphases.

One aspect of freebody analysis does have general applicability: the notion of a system (and surroundings). We stress this important concept in the context of energy, where it plays a key role (and where it receives inadequate attention in traditional mechanics courses). We also emphasize the concept of system in collisions involving conservation of momentum and of angular momentum.

B. Energy

The second fundamental principle introduced is the energy principle. The book's central focus is on the very careful and precise specification of system and surroundings, which is required to be able to apply the energy principle,

$$\Delta E_{\mathrm{system}} = W + Q, \tag{3}$$

in arbitrarily complex situations. The distinction between work (due to forces exerted by objects external to the system) and potential energy (associated with pairs of interacting objects within a system) is carefully made, including how these terms appear to the left or the right of the equals sign depending on the choice of system.[29]

We are now in a position to use two fundamental principles simultaneously in an analysis, reinforcing the distinction between them. The simplest applications involve adding energy calculations to previously done computational problems, both to check the accuracy of computations, and to examine the forms of energy in interacting systems.

The contemporary, microscopic emphasis of the course makes it natural to include problems involving a change of particle identity (for example, neutron decay) to make it clear that there is no conservation principle for mass. We have been encouraged to see students often include rest energies in energy calculations even when they know $\Delta(mc^2)$ is zero and the rest energies cancel out, indicating an awareness of the possibility that the rest energies might change.[30]

We make contact with what students have learned in their chemistry courses and introduce the most immediately accessible aspects of quantum mechanics: energy levels and transitions between energy levels associated with photon emission or absorption. Our intent is to introduce some key elements of the quantum picture to make sure that students encounter the idea that classical mechanics is not a closed, universal theory but only an important approximation, valid for macroscopic systems. Students work with systems with quantized energy levels, including the hydrogen atom and the quantum harmonic oscillator (with application to diatomic molecules).

Several aspects of the treatment of energy may be more striking to experts than they are to the novices in the beginning course. We report these briefly here.

Building on intuitively and kinesthetically plausible notions (energy is associated with motion; exerting a force through a distance requires the expenditure of something; you can't get something for nothing), the energy of a particle is introduced by finding a function of the speed whose change is equal to the work. In one dimension, we have

$$dE = F\,dx = \frac{dp}{dt}\,dx, \tag{4}$$

$$\frac{dE}{dx} = \frac{dp}{dt}. \tag{5}$$

In other words, the spatial derivative of the particle energy is equal to the time derivative of the (relativistic) particle momentum. (One advantage of this approach is the use of antiderivatives rather than integrals at a time when some students have not yet learned about integration in their calculus courses.) A function that satisfies this relation is

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}. \tag{6}$$

We show that the separation of particle energy into rest energy plus kinetic energy is useful, and develop the low-speed approximation to the kinetic energy, furnishing another opportunity to discuss approximations and range of validity.

In the usual way, the concept of energy is generalized to multiparticle systems by defining potential energy as the negative of the work done on the particles by forces internal to the system, and we state the energy principle:

$$\Delta E_{\mathrm{system}} = W_{\mathrm{ext}} + Q, \tag{7}$$

where Q is the energy transfer due to a temperature difference between the surroundings and the system. We include thermal energy in the analyses very early, in order to integrate mechanics and thermal physics.[31] The macroscopic thermal energy of a solid is identified with the atomic-level kinetic and potential energy of the ball-and-spring model.

A significant innovation involves the zero of potential energy. In the nonrelativistic context, traditionally presented to students, potential energy has an arbitrary additive constant, and only differences in potential energy are physically meaningful. It is normally considered to be a mere convenience that $1/r$ potential energies are adjusted to go to zero at infinity. However, in the contemporary relativistic context, potential energy *must* go to zero at infinity. For example, consider a positron and electron each of mass m initially at rest far from each other. The (relativistic) energy of the two-particle system is $2mc^2$, with no additive constant possible (otherwise the four-momentum would not transform properly). Because of this constraint, the electric potential energy must go to zero at infinite separation.[32]

There are pedagogical advantages to invoking the absolute character of potential energy in the relativistic context. Students are often uneasy about negative energies such as are encountered with attractive forces. In the larger relativistic context the energy of a system is never negative. Rather, the energy of a multiparticle system can be either more or less than the sum of the rest energies of the individual particles, but always positive.

C. Applying the momentum and energy principles to multiparticle systems

With two fundamental principles, we are now in a position to attempt a deeper analysis of multiparticle systems. Expressing the kinetic energy as translational kinetic energy (associated with the motion of the center of mass) plus kinetic energy relative to the center of mass, introduces a concept that is important in later physics courses and in physical chemistry courses. To sustain a careful and precise treatment of energy for multiparticle systems, the concept of the ''point-particle system'' is introduced, as described in Ref. 3.

Both relativistic and nonrelativistic collisions are analyzed using the momentum and energy principles. Rutherford scattering is analyzed in some detail, and the students write a computer program to study how the impact parameter affects the scattering angle (the recoil of the gold nucleus is part of the model). Because the students have been working with momentum for many weeks before this study of collisions, they are applying familiar concepts to unfamiliar processes rather than applying an unfamiliar momentum concept to an unfamiliar process. Traditionally, momentum is not introduced until late in the course, requiring students to digest two new concepts at the same time (collisions and momentum) and leaving little time to learn the momentum concept well.

D. Angular momentum

The final fundamental principle, the angular momentum principle, is introduced in its full form, including the separation into translational (orbital) and rotational (spin) angular momenta. The initial emphasis is on systems subjected to zero torque, where the angular momentum of the system doesn't change, especially in collisions. We find these processes significantly easier for the students to deal with than processes where a torque changes the angular momentum. The quantization of angular momentum is discussed in this chapter, with application to the Bohr model of the hydrogen atom.

At this point in the course, students can analyze situations where all three major mechanics principles may come into play in one problem: momentum, energy, and angular momentum. Because at this point students have worked extensively with momentum, they rarely confuse angular momentum with linear momentum. We find that the introduction of angular momentum significantly sharpens their understanding of the nature of a fundamental principle, one which is very widely or even universally applicable. At this point even strong students may ask in wonderment, ''Does the momentum principle really apply even to a rotating system?''

E. Thermal physics

While thermal physics has been entwined with other aspects throughout the course, the final portion of our modern mechanics course deals more deeply with thermal topics. Following the suggestions of Moore and Schroeder,[33] students write simple programs to calculate the entropy of an Einstein solid (a ball-and-spring model in which each atom is modeled as three isolated quantum oscillators in x, y, and z), its temperature as a function of energy, and its heat capacity as a function of temperature. The ball-and-spring story line leading to this activity was discussed in the section on macro–micro homework problems. Again as suggested by Moore and Schroeder, an additional step in the analysis leads to the concept of the Boltzmann factor, which is applied to understanding thermal aspects of diatomic gases.

Kinetic theory is an additional application of the concepts of momentum and collisions, with connections to the previous section on statistical mechanics. A final chapter deals with heat engines, as an application of the entropy principle (second law of thermodynamics). Analyzing heat engines by using the concept of entropy is much preferable to the usual sequence, in which entropy is defined by considering a heat engine whose working substance is an ideal gas, a procedure that is too abstract to be meaningful and which encourages the incorrect notion that an ideal gas is essential for a heat engine to function.

Additional discussion of the thermal aspects of the course may be found in Ref. 3.

IV. THE INTERMEDIATE MECHANICS COURSE

The broader foundation laid by this introductory mechanics course could provide the impetus and rationale for restructuring the intermediate mechanics course taken by third-year physics majors. The goals usually stated for the intermediate mechanics course are to introduce a higher level of abstraction, to give students practice in more complex problem solving with increased depth in mathematical physics, and to provide a foundation for fourth-year quantum mechanics, especially through the introduction of Lagrangian and Hamiltonian viewpoints. We believe these goals to be too narrow, and refer to our previously stated goals for introductory mechanics in considering the content and approach of the intermediate course.

The higher level of abstraction attained in the intermediate mechanics course should be strongly coupled to the idea that all reasoning in mechanics is based on a small number of powerful fundamental principles. This minimalism can unfortunately be missed by students struggling to master the spate of new mathematical techniques that traditionally fill this course, yet it may be the most important idea we can teach here. In reformulating fundamental principles more abstractly (as in the Lagrangian and Hamiltonian formulations), there is an opportunity to explore this minimalism at a higher level. Edwin Taylor[34] has urged cogently that we pursue this goal by beginning with the principle of least action, from which all other formulations can be derived; alternatively, this principle might be the punch line of a restructured course.

Modeling the real world should play a major role. Adopting this focus does not necessarily change the list of topics to be covered, but it does require that a significant number of larger, real-world problems be integrated into the course. Real systems, both macroscopic and microscopic, in all their messy complexity, need to be the focus of a significant fraction of our analyses at the intermediate level. Idealizing complex systems and making assumptions and approximations are no less important at the intermediate level; these thinking skills can be developed only in the context of situations involving specific objects and materials instead of generic objects of mass *m*. Computational modeling should be integrated here as well, because it offers a venue for observing the evolution of a system predicted or described by the formal methods, and for applying computationally cumbersome concepts developed in the course (such as rotation matrices and moment of inertia tensors).

Connections to modern physics, especially to quantum mechanics (to which students have typically been introduced in their sophomore-level courses), should be made explicit in this course, and the rationale for and value of semiclassical analyses should be explored as another example of the broad applicability of fundamental principles. Physics education research frequently reminds us of a fact we would be pleased to forget: students generally do not pick up important ideas by osmosis; explicit instruction is required to make these connections between formal classical mechanics and other domains.

[a]Electronic mail: rwchabay@unity.ncsu.edu

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[7]Reference 2, Problem 3.9, p. 110.
[8]Reference 2, Problem 6.4, p. 228.
[9]Reference 2, Problem 10.1-10.4, pp. 378–379.
[10]Reference 2, Problem 4.15, p. 161.
[11]Reference 2, Problem 8.10, p. 291.
[12]Reference 2, Problem 4.16, p. 161.
[13]Reference 2, Problem 2.11, p. 70.
[14]Reference 2, Problem 2.17, p. 72.
[15]Reference 2, Problem 8.2, p. 289.
[16]Reference 2, Problem 10.15, p. 381.
[17]The textbook is written in such a way that any computing environment preferred by the instructor can be used, but we recommend the use of the VPython language/programming environment. VPython is a particularly appropriate tool because of its support of vector computations and three-dimensional visualization. See ⟨http://vpython.org⟩. VPython is free, multi-platform, and open source.
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[27]T. O'Brien Pride, S. Vokos, and L. C. McDermott, ''The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems,'' Am. J. Phys. **66**, 147–157 (1998).
[28]B. Sherwood and R. Chabay, ''Predicting effects of instruction: Distinguishing *P* from *K* and *L*,'' AAPT Announcer **31** (4), 103 (2001).
[29]B. Sherwood, ''Pseudowork and real work,'' Am. J. Phys. **51**, 597–602 (1983). Also see B. Sherwood and W. Bernard, ''Work and heat transfer in the presence of sliding friction,'' *ibid.* **52**, 1001–1007 (1984).
[30]Although students work with relativistic momentum and energy, relativistic kinematics is left to a subsequent modern physics course. However, in a longer version of the course offered to physics majors at Purdue University, Mark Haugan weaves relativistic kinematics into his teaching of modern mechanics. See M. Haugan, ''Preempting student difficulties with special relativity,'' AAPT Announcer **32** (2), 134 (2002).
[31]Reference 3.
[32]Fred Reif (private communication) has pointed out that the elimination of the additive constant for potential energy because of the absolute rest energy associated with a particle is analogous to the elimination of the classical arbitrary additive constant for entropy because of the existence of lowest-energy ground states; entropy *must* go to zero at absolute zero.
[33]T. Moore and D. Schroeder, ''A different approach to introducing statistical mechanics,'' Am. J. Phys. **65**, 26–36 (1997). A specific realization of these ideas is found in the statistical mechanics section of the textbook by T. Moore, *Six Ideas that Shaped Physics/Unit T: Some Processes are Irreversible* (WCB/McGraw-Hill, Boston, 2003).
[34]E. Taylor, ''A call to action,'' Am. J. Phys. **71**, 423–425 (2003).