

A MORE COHERENT TOPIC SEQUENCE FOR E&M

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The traditional sequence of topics in the E&M portion of the calculus-based introductory physics poses serious conceptual problems for the student. New concepts are introduced extremely rapidly; abstractions are layered on top of abstractions; complex mathematical formalisms are introduced rapidly; and the student is expected to assimilate all of this quickly. To many students it is not even clear what phenomena this elaborate formalism is designed to explain. Further, topics are presented in an intrinsically fragmented way. The concepts of charge, electric field, and flux, introduced in electrostatics, are abandoned entirely in the treatment of circuits, which deals with current and potential. Magnetic field is introduced very late. At the end of the traditional course, it is not uncommon for students still to be confused about the difference between charge and field. We have addressed these issues by making two fundamental changes in the curriculum. First, we have emphasized an intermediate level of abstraction, focusing on field as a primary concept at an intermediate level of abstraction, and a coherent thread throughout the course, including circuits. Second, we have altered the sequence of topics, introducing magnetic fields early, to strengthen students' comprehension of the field concept; and we have delayed topologically and mathematically complex topics such as Gauss's law until late in the course, when students have accumulated extensive experience with 3D patterns of field in space and with the charge distributions responsible for these fields. In the traditional sequence magnetic field and magnetic force are introduced quite late in the course, when students (and instructors) are tired and have little remaining time to gain adequate experience with the topic, or to compare and contrast electricity and magnetism. Early introduction of magnetic field bolsters student understanding of the field concept, because the analysis of magnetic interactions necessarily requires the two-step process of thinking about fields and then forces. Faraday's law is more difficult when student experience with magnetic field has been quite brief, and when the flux concept has already been forgotten, whereas in our curriculum the juxtaposition of Gauss's law and Faraday's law reinforces the concept of flux, and students have worked with magnetic fields and forces for many weeks before encountering Faraday's law. See <http://www4.ncsu.edu/~rwchabay/mi>.

Why E&M is difficult for students

Traditionally students are introduced to electricity and magnetism (E&M) in the second half of the introductory calculus-based physics course, after they have completed an introduction to classical mechanics. Even students who have done well in the first part of the course often see E&M as difficult and confusing.

In E&M, students suddenly encounter for the first time a level of abstraction and mathematical sophistication far beyond what they experienced in mechanics. In mechanics, many situations involved familiar macroscopic objects: balls and sticks, cars and airplanes. At least some important concepts, such as velocity and force, were easily related to everyday experience. In E&M the student is suddenly introduced to a world in which almost all of the things under consideration are invisible; they are either microscopic, like electrons, or abstractions, such as field, flux, and potential. Integral calculus becomes a central mathematical tool, and students are asked to apply it in unfamiliar ways, such as calculating the path integral or surface integral of a quantity expressed as a vector dot product.

In the traditional introductory E&M sequence, this conceptual and mathematical complexity is exacerbated by the extraordinarily rapid introduction of a long sequence of new concepts. The ideas of charge, electric force, field, flux, and Gauss's law are often presented within the first week of the course. These are quickly followed by the concepts of potential, potential difference, and electric current. Students may be overwhelmed by this rapid piling up of abstract ideas, and usually are not given sufficient practice to be able to apply these concepts

reliably, nor to discriminate them from each other. Indeed, it is not uncommon to hear instructors complain, at the end of a semester, that a significant fraction of their students still do not know the difference between charge and field. By the end of the course, it is not unusual for good students to have forgotten the expression for the electric field of a single point charge, since this has not been used for many weeks.

Goals of the introductory E&M sequence

Some research and development in physics education has focused on remedying particular problems with this sequence, by giving students additional focused practice on one concept or another. We have chosen instead to re-examine the intellectual structure of the E&M curriculum, in an attempt to identify which concepts are centrally important, how these concepts are related, and how they may be introduced to students in a coherent, comprehensible sequence. Our overall goals are those of the *Matter & Interactions* curriculum (Chabay & Sherwood, 2002), which is discussed elsewhere in this volume. The goal of the *Matter & Interactions* (M&I) curriculum is to engage students in the contemporary physics enterprise, by emphasizing:

- A small number of fundamental principles, from which students start analyses
- The atomic nature of matter, and macro/micro connections
- Unification of topics, facilitated by the atomic view of matter
- Modeling physical systems, including computational modeling

Field: An intermediate level of abstraction

The concept of field is central to electricity and magnetism. However, in the traditional introductory course, this concept is not used at all during large sections of the course, including those sections dealing with electric circuits and with Faraday's law. Consequently, the field concept does not appear central to students, who are kept busy plugging numbers into formulas for equivalent resistance and mutual inductance. Our goal, in the redesign of the E&M sequence, was to make the field concept appear both important and useful to the students in the course.

The field concept is a significantly more abstract concept than any of the quantities typically used in mechanics. The effort required to understand and become comfortable in using this concept is, however, justified by the immense gain in predictive and explanatory power it affords. In particular, the classical model of electromagnetic radiation is truly incomprehensible to a student who has not mastered the concepts of electric and magnetic fields. Like the Newtonian synthesis in mechanics, Maxwell's equations and his explanation of the nature of light are one of the crowning intellectual achievements of classical physics. Introductory students are capable of understanding this triumph if and only if they have had sufficient practice and sufficiently varied experiences with the concepts of electric and magnetic fields, and their effects on matter, before they reach this complex idea. For this reason, and because the field concept alone affords students the opportunity to gain significantly in intellectual sophistication, we have made the field concept the central focus of the introductory E&M sequence, and the backbone of the story told in the course.

In the remainder of this paper we discuss details of an altered sequence of topics designed to focus on the field concept, including the late introduction of Gauss's law and the early introduction of magnetic field.

Gauss's law

In the traditional sequence Gauss's law is introduced very early, sometimes during the first week of the course. Generations of physics teachers have lamented the fact that the students just don't understand Gauss's law. But from a cognitive point of view it is quite clear why this happens despite the best efforts of good teachers. At this early stage of the course, students are struggling with what is for many of them a subtle distinction between charge and field, yet Gauss's law embodies a complex topological relationship between charge and patterns of field in three-dimensional space. Early in the course students have had no experience with the kinds of patterns of field that are possible in space, but these patterns of field lie at the heart of the topological relationship. Surface integrals are typically unfamiliar to students, so flux is a challenging concept. Moreover, students have had little experience visualizing 3D geometries, nor have they had prior experience with symmetry arguments of the kind that play a large role in the actual applications made of Gauss's law. All these multiple conceptual problems and barriers make introducing Gauss's law early in the course a pedagogical disaster.

Why is Gauss's law introduced early? The real justification is that it is needed to prove three important properties of electrostatics, that charge is found only on the surface of a conductor (not in the interior), that the electric field inside a conductor (and inside an empty cavity in a conductor) in static equilibrium is zero everywhere, and that the field of a uniformly charged spherical shell is zero inside and point-like outside. However, early in the course students are not able to understand the elegant proofs constructed with Gauss's law! Probably the reason Gauss's law has traditionally been treated early is to satisfy the desire for rigor on the part of the teacher. Unfortunately, at this point it is only the instructor who is able to appreciate this rigor.

Because it poses such conceptual challenges for students, we have delayed the introduction of Gauss's law until much later in the course. How do we introduce the properties enumerated above? We make all three results plausible, and state that we will later prove them rigorously, when we discuss Gauss's law. Students are quite willing to accept this approach.

Because we have emphasized the microscopic structure of conducting materials, we are able to prove by contradiction that if there were a nonzero electric field the mobile charges would move, which contradicts the assumption of static equilibrium. (This argument is not sufficient to prove that the electric field in an empty cavity within a conductor in static equilibrium is zero.)

The electric field of a uniformly charged spherical shell can be derived by brute force using calculus. We show how to set up the integral over the charge distribution but explain that an easier route to the final result will be available later using Gauss's law. Gauss's law is often used to obtain the electric field of an infinite uniformly charged plate and an infinite uniformly charged rod, but we obtain these results from limiting cases of the finite disk and finite rod, which are derived fairly easily by integration. There is a pedagogical advantage to emphasizing large but finite charge distributions rather than infinite ones, since the latter can raise awkward conceptual issues. (For example, a student correctly pointed out that it would take infinite time to charge a capacitor with infinite plates.)

In M&I, Gauss's law is introduced about two thirds of the way through the course, at the point where students have had a great deal of experience with patterns of electric (and

magnetic) field in three-dimensional space. We exploit this familiarity to introduce Gauss's law first in a qualitative, visual form that emphasizes what students already know about patterns of field in space and the distributions of charge responsible for them. Qualitatively, using dynamic 3D computer visualization software (supported by static figures in the textbook), we show that there is a relationship between the amount of charge inside a closed 3D surface and the surface integral of field on that surface. Only after making the topological connection plausible do we prove the formal quantitative version of Gauss's law. We then use Gauss's law to prove that there cannot be any nonzero charge density in the interior of a conductor, and that the electric field inside a conductor in static equilibrium (and inside an empty cavity) must be zero. We also do the usual applications of Gauss's law to the field of a sphere, rod, and plate. Our experience is that students are able to appreciate the nature of Gauss's law when presented late in the course, and can use Gauss's law to relate charge and field. Ampere's law, which, like Gauss's law, involves patterns of field in space, is treated in the same chapter, after a discussion of Gauss's law for magnetism and the apparent absence of magnetic monopoles.

Magnetic field

In the traditional sequence magnetic field and magnetic force are introduced quite late in the course, after electrostatics and circuits. There are many disadvantages to this delay. A purely pragmatic issue is that late in the course students are tired, and unenthusiastic about confronting a challenging new concept. If magnetism is introduced very late, students have little remaining time in the course to gain adequate experience with the topic, and little time to compare and contrast electricity and magnetism. Faraday's law is made more difficult when students' experience with magnetic field has been quite brief.

Perhaps more seriously, experience with magnetic field and magnetic force is necessary to solidify students' understanding of the concept of field. Even strong students are frequently disconcerted when, late in the course, they find themselves struggling with the concept of field in this new context. We surmise that in the analysis of electric interactions it is possible for students to imagine distributions of source charges interacting directly with other charges, instead of focusing on field as a mediator of the interaction. However, the complex spatial nature of magnetic interactions leaves no alternative to the two-step process of determining the magnetic field due to the source charges, then using this field to find the direction and magnitude of magnetic forces.

Besides strengthening students' understanding of fields, the early introduction of magnetic field allows students to use the magnetic field of moving charges as a probe of current in electric circuits. Initially, we view magnetic field operationally as a field that affects a compass. It is produced by moving charges, and students observe the magnetic effects that currents in simple circuits have on a nearby compass. They observe the magnetic field near a coil and a bar magnet and identify a dipole-like pattern familiar from earlier work with electric dipoles. An atomic model allows students to predict the magnetic dipole moment of a bar magnet, and confirm their prediction by using a compass to measure it.

In the traditional sequence electric field is dropped once circuits are introduced, and magnetic field is not introduced until after the study of circuits is completed. In contrast, in M&I both electric and magnetic field remain central throughout the subsequent chapters on circuits. The treatment of circuits, discussed below, focuses on microscopic description and analysis. The deep sense of microscopic mechanism enriches the understanding of Kirchhoff's loop and

node rules, and provides additional practice with electric and magnetic fields and their relationship to each other. The chapter on magnetic force also makes strong connections to electric field and force, and to modern physics. In particular, a simple example of two protons repelling each other, viewed from two different reference frames, shows how electric and magnetic effects are dependent on the choice of reference frame, and offers the opportunity to explore the idea that time runs differently in different reference frames.

As a result of this altered sequence, students have many weeks of experience with magnetic field, and many weeks of experience in comparing and contrasting electric field and magnetic field, including engaging in frequent tasks involving both kinds of field.

Faraday's law

Faraday's law is usually difficult for students. It involves a dynamic connection between magnetic and electric phenomena, introduced at a time when students have had only a rather brief exposure to magnetic field. Moreover, the integral form uses the concept of flux, which is typically introduced at the start of the course in the context of Gauss's law but not used again until much later, with the introduction of Faraday's law. The effect is to use a forgotten concept (flux) to relate a line integral of electric field (emf) to the time derivative of the surface integral involving a quantity with which the students have had inadequate practice (magnetic field). It is not surprising that Faraday's law is usually difficult for students.

With the new E&M sequence Faraday's law is surprisingly easy. It is introduced with emphasis on the curly electric field that is found surrounding a region of time-varying magnetic field. Students at this point in the course have had lengthy experience with both electric field and magnetic field, including their patterns in space. The flux concept is fresh in their minds, because Gauss's law is treated immediately preceding Faraday's law. The combination of long-term experience with magnetic field and the just-in-time teaching of the flux concept makes Faraday's law quite accessible to students.

While we have found it completely feasible to introduce magnetism early and use it repeatedly, it would be difficult to find appropriate applications of the flux concept to sustain and build student facility with the concept, even if it were possible to teach Gauss's law at the start of the course. Hence there is another advantage to introducing the flux concept just in time, immediately preceding Faraday's law. It is worth noting that we discuss motional emf in the chapter on magnetic force, well *before* introducing Faraday's law. This helps make an important distinction between these two very different mechanisms for producing emf, whereas in the traditional sequence the two effects are often not clearly delineated.

Electromagnetic radiation

After dealing with Gauss's law for electricity and magnetism, Ampere's law, and Faraday's law, we are ready to consider Maxwell's extension to Ampere's law and show that crossed electric and magnetic fields can propagate in empty space, at the speed of light. With diagrams showing the effects of retardation we make it plausible that an accelerated charge would produce radiation, and we state without proof the formula for the fields of an accelerated charge, so we have a mechanism for the production of radiation. This sense of mechanism makes accessible the classical interaction of electromagnetic fields with matter. The course all along has emphasized the effect of fields on the charged particles of which matter is composed, so it is easy and natural to talk about the acceleration of electrons in

matter by the electric field in incident radiation, and subsequent re-radiation by these accelerated electrons. This view brings to physical optics a clear sense of mechanism. Moreover, because the preceding mechanics course includes discussion of photon emission and absorption, we are now in a good position to discuss wave-particle duality.

Unifying electrostatics and circuits

One important aspect of the M&I curriculum involves the unification of topics traditionally presented as unrelated. In the traditional introductory E&M course electrostatics and circuit phenomena are treated as nearly completely separate subjects. Electrostatics is discussed in terms of charge and field, but circuits are discussed exclusively in terms of current and potential, and the concept of electric field is dropped from view, never to return.

Inspired by Haertel (1987), we first engage students in the analysis of circuits from a microscopic point of view. Ohmic materials are described in terms of the microscopic relation $v = uE$, where v is the drift speed, u is the mobility of the mobile charges, and E is the electric field. The field inside circuit wires is shown to be due not only to charges in and on the battery but also to those on the surfaces of the circuit elements. Students learn to analyze both DC and RC circuits directly in terms of the Coulomb interaction and the atomic nature of matter. This analysis in terms of charge and field makes a strong connection with electrostatics, unifying the two topics, and it provides a strong sense of mechanism for the behavior of simple circuits.

Overview of the new sequence

- Stationary charges (4 weeks)
 - Electric field; effect on matter; field of distributed charges; potential
- Moving charges (5 weeks)
 - Magnetic field; microscopic view of circuits; macroscopic view
 - Magnetic force, including motional emf
- Reasoning about patterns of field in space (1 week)
 - Flux and Gauss's law; Ampere's law
- Time-varying fields, accelerated charges (4 weeks)
 - Faraday's law: curly electric field associated with time-varying magnetic field; induced emf
 - Ampere-Maxwell law; electromagnetic radiation; radiation by accelerated charges
 - Physical optics; wave-particle duality

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References

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